Linear Guidance Laws for Space Missions

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A general analysis of guidance laws is presented along with a review of the propagation of state perturbations. Linearized guidance laws are derived for fixed and variable time of arrival, and for the Shuttle phasing and height guidance maneuvers. In general, six interrelated partials are involved in each guidance law. These partials relate the maneuver delta velocity and time of flight to perturbations in initial position, velocity, and time. The partials can be expressed as a function of a single fundamental guidance partial, which relates the maneuver to the final position perturbation, and the partials from the upper row of the augmented time transition matrix (augmented with the dynamical state vector). Guidance can be interpreted as a form of offset targeting.

Introduction

SPACE trajectories involve a series of coast phases separated by maneuvers. The maneuvers can be derived by imposing sufficient guidance constraints to uniquely define the maneuver. The guidance constraints can be imposed on the maneuver, postmaneuver trajectory, or some combination thereof. Impulsive maneuvers may be assumed for many space missions. The impulsive maneuvers can be computed exactly or with a linearized model. The linearized model could be used for real-time software because trajectory perturbations are generally small, but a more likely use is in preflight covariance analysis. The linear model contains guidance matrices for updating through the maneuvers, and transition matrices for updating through the coasting phases. This paper presents two general techniques for solving the linear guidance problem.

Most applications of linearized guidance laws have been based on fixed- and variable-time-of-arrival guidance.[1-4] Many other linearized laws can be introduced by imposing different constraints on the maneuver and postmaneuver trajectory.[5,6] Some general properties of linearized guidance laws were established by Cicolani.[7] A general solution to the linear guidance problem was proposed by Tempelman[5] by introducing a generalized linear constraint equation. This paper attempts to define some general techniques for establishing the relationship between the maneuver and the perturbations in position, velocity, and time using a more intuitive approach than presented in Ref. 5. Fixed- and variable-time-of-arrival guidance laws are reviewed to demonstrate the techniques and to serve as an introduction to more complicated maneuvers, such as the Shuttle phasing and height maneuvers.

Conic State Propagation and Properties of Conic Transition Matrices

A linearized model of conic motion can be defined based on the time-transition matrix Φ (generally called the state transition matrix) for propagating perturbations through the nominal flight time, modified by an additional term to allow for a time perturbation δt in the flight time

$$\delta X_f = \Phi \delta X_i + D_f \delta t = \frac{\partial X_f}{\partial X_i}\delta X_i + \frac{\partial X_f}{\partial t}\delta t$$

$$\begin{bmatrix} \delta R_f \\ \delta V_f \end{bmatrix} = \Phi \begin{bmatrix} \delta R_i \\ \delta V_i \end{bmatrix} + \begin{bmatrix} V_f \\ A_f \end{bmatrix} \delta t, \quad \Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_3 & \Phi_4 \end{bmatrix} \tag{1}$$

where X is the six-dimensional state (R, V) composed of position and velocity, D represents the dynamical state vector (V,A) composed of velocity and acceleration, and the subscripts i and f refer to the initial and final points on the trajectory. Equation (1) can be represented in a seven-dimensional matrix-vector format

$$\begin{bmatrix} \delta X_f \\ \delta t \end{bmatrix} = \begin{bmatrix} \Phi & D_f \\ 0_6^T & 1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta t \end{bmatrix} = \Phi_A \begin{bmatrix} \delta X_i \\ \delta t \end{bmatrix} \tag{2}$$

where δt is the change in flight time, 0_6 a six-dimensional null vector, and Φ_A is considered a (7×7) augmented time transition matrix. If the final perturbation is to be defined with respect to a target trajectory (subscript t), Eq. (2) can be expressed as

$$\begin{bmatrix} \delta X_f \\ \delta t \end{bmatrix} = \begin{bmatrix} \Phi & D_{fr} \\ 0_6^T & 1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta t \end{bmatrix} = \Phi_{AR} \begin{bmatrix} \delta X_i \\ \delta t \end{bmatrix} \tag{3}$$

where

$$D_{fr} = D_f - D_t = \begin{bmatrix} V_{fr} \\ A_{fr} \end{bmatrix} = \begin{bmatrix} V_f - V_t \\ A_f - A_t \end{bmatrix}$$

Φ_{AR} is considered a (7×7) augmented relative time transition matrix and the subscript fr represents final relative.

The change in flight time δt in Eq. (2) is defined by differentiating the time of flight t, equal to the final time t_f minus the initial time t_i

$$\delta t = \frac{\partial t}{\partial t_f}\delta t_f + \frac{\partial t}{\partial t_i}\delta t_i = \delta t_f - \delta t_i \tag{4}$$

When the change in time of flight δt is zero, δt_f equals δt_i. The propagation of state errors is obtained by combining Eqs. (1) and (4)

$$\delta X_f = \Phi \delta X_i + D_f(\delta t_f - \delta t_i) \tag{5}$$

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The relationship between initial time and state perturbations in position and velocity can be established by taking the total differential of any vector (or scalar) function $Y(X_i, t_i)$

$$\delta Y = \frac{\partial Y}{\partial X_i}\delta X_i + \frac{\partial Y}{\partial t_i}\delta t_i \tag{6}$$

Setting δY equal to zero and dividing through by δt_i yields the "equivalence relationship" between the partials involving initial time perturbations and initial state perturbations

$$\frac{\partial Y}{\partial t_i} = -\frac{\partial Y}{\partial X_i} D_i \tag{7}$$

The initial dynamical state vector may be updated with the time transition matrix[8]

$$D_f = \Phi D_i \tag{8}$$

This equation does not seem to follow rigorously by setting Y equal to X_f in Eq. (7).

Assuming an update to the final time t_f (i.e., $\delta t_f = 0$), Eqs. (5) and (8) can be combined to give

$$\delta X_f = \Phi \delta X_i - D_f \delta t_i = \Phi(\delta X_i - D_i \delta t_i) \tag{9}$$

The update of the initial state error δX_i, δt_i to time t_f can therefore be accomplished in two ways. One way is to update the initial state perturbation δX_i by an amount obtained by multiplying the initial dynamical state D_i by $-\delta t_i$ (the minus sign is because δt equals $-\delta t_i$ when δt_f is zero), and then propagating the resulting perturbed state through the transition matrix Φ to the final point (Fig. 1a). The other way is to update the perturbed state δX_i through the flight time, and then update an additional amount obtained by multiplying the final dynamical state D_f by $-\delta t_i$ (Fig. 1b). It can therefore be concluded that for linear problems an initial time error can be converted into an initial state error in position and velocity using

$$\delta X_i = -D_i \delta t_i \tag{10}$$

The time transition matrix Φ, a symplectic matrix,[9] can be expressed in terms of the derivatives of the submatrices

$$\Phi = \begin{bmatrix} G_G^{-1}\dfrac{\partial \Phi_3}{\partial t} & G_G^{-1}\dfrac{\partial \Phi_4}{\partial t} \\ \dfrac{\partial \Phi_1}{\partial t} & \dfrac{\partial \Phi_2}{\partial t} \end{bmatrix} \tag{11}$$

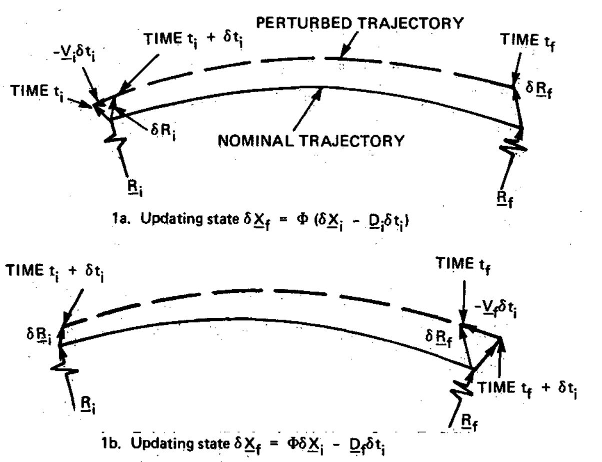

Fig. 1 Different ways of updating the state perturbation in position and time to time t_f.

where G_G is the gravity-gradient matrix. The following symmetric matrices can be derived using the symplectic property of Φ

$$\Phi_1\Phi_2^T, \quad \Phi_3\Phi_4^T, \quad \Phi_4^T\Phi_2, \quad \Phi_3^T\Phi_1 \tag{12}$$

The submatrices of Φ satisfy

$$\Phi_1\Phi_4^T - \Phi_2\Phi_3^T = I_3 \tag{13}$$

I_3 is a 3×3 identity matrix.

Equation (13) demonstrates that only three of the time transition submatrices are independent.

There are other formulations of the time transition matrix,[8] some of which are of special interest for guidance problems. The most important of these is the matrix that relates velocity perturbations to position perturbations based on constant time of flight

$$\begin{bmatrix} \delta V_i \\ \delta V_f \end{bmatrix} = \begin{bmatrix} L_1 & L_2 \\ L_3 & L_4 \end{bmatrix} \begin{bmatrix} \delta R_i \\ \delta R_f \end{bmatrix} = L \begin{bmatrix} \delta R_i \\ \delta R_f \end{bmatrix} \tag{14}$$

The matrix L is generally referred to as the Lambert transition matrix because of the use of Eq. (14) in solving the fixed-time-of-arrival guidance problem. Using the definition of the time transition matrix, the Lambert matrix can be expressed as

$$L = \begin{bmatrix} -\Phi_2^{-1}\Phi_1 & \Phi_2^{-1} \\ -\Phi_2^{-T} & \Phi_4\Phi_2^{-1} \end{bmatrix} \tag{15}$$

where the superscript $-T$ indicates the transpose of the inverse. The inverse of the Lambert matrix is

$$L^{-1} = \begin{bmatrix} -\Phi_3^{-1}\Phi_4 & \Phi_3^{-1} \\ -\Phi_3^{-T} & \Phi_1\Phi_3^{-1} \end{bmatrix} \tag{16}$$

The diagonal submatrices in L and L^{-1} are symmetric [verified from the symmetries of the matrices in Eq. (12)]. The Lambert matrices can be used to propagate the dynamical elements of velocity and acceleration

$$\begin{bmatrix} A_i \\ A_f \end{bmatrix} = L \begin{bmatrix} V_i \\ V_f \end{bmatrix}, \quad \begin{bmatrix} V_i \\ V_f \end{bmatrix} = L^{-1} \begin{bmatrix} A_i \\ A_f \end{bmatrix} \tag{17}$$

Cutoff conditions other than time can be used to terminate the trajectory.[8] Examples are central angle, flight-path angle and velocity, and a specified direction in position space that should contain the final perturbed position vector. For example, if a geometric constraint is imposed at the final point in the form

$$K^T \delta X_f = 0 \tag{18}$$

where K is a six-dimensional cutoff constraint vector, then combining Eqs. (1) and (18), solving for δt, and inserting into Eq. (1) results in the seven-dimensional transition matrix Φ_{K7}

$$\Phi_{K7} = \begin{bmatrix} \Phi_K & 0_6 \\ T^T & 1 \end{bmatrix} \tag{19}$$

where

$$\Phi_K = \Phi + D_{fr} T^T$$

$$T = \frac{\partial t}{\partial X_i} = -\frac{\Phi^T K}{K^T D_{fr}}$$

Guidance laws that involve updates not based on time will generally be formulated in terms of the Φ_K submatrices.

The Linear Guidance Problem

The linear guidance problem involves the determination of a maneuver that compensates for the initial state perturbations in position, velocity, and time. There are six conditions required to establish a conic trajectory. The initial perturbed position vector, $\boldsymbol{R}_i+\delta\boldsymbol{R}_i$ (assuming an instantaneous maneuver), determines three conditions. Three other conditions must be imposed by specifying guidance constraints to allow the determination of the three components of the postmaneuver velocity. These conditions can be imposed on the maneuver and/or the transfer trajectory following the maneuver. Examples of these conditions are scalar quantities which are to be held fixed between the initial and terminal points (such as time of flight or central angle), a scalar direction in which the final perturbed position is to occur, a specified maneuver direction, and the constraint that the maneuver is to be minimized.

The maneuver is defined as the difference between the postmaneuver and premaneuver velocity vectors

$$\delta\boldsymbol{V}_m=\boldsymbol{V}_i^+-\boldsymbol{V}_i=\boldsymbol{V}_i^+-\boldsymbol{V}_{ni}-(\boldsymbol{V}_i-\boldsymbol{V}_{ni})=\delta\boldsymbol{V}_i^+-\delta\boldsymbol{V}_i$$

$$\delta\boldsymbol{V}_i^+=\delta\boldsymbol{V}_m+\delta\boldsymbol{V}_i \tag{20}$$

where $\boldsymbol{V}_i^+$ is the velocity vector after the maneuver and $\boldsymbol{V}_{ni}$ is the nominal velocity at the initial position. The + superscript indicates postmaneuver trajectory conditions. In order to use the augmented time transition matrix Φ_A to update the state following the maneuver, the guidance calculation should insert the final time perturbation δt_f, if one exists, into the state perturbation vector following the maneuver. The perturbed state $(\delta\boldsymbol{R}_i,\delta\boldsymbol{V}_i)$ used in Eqs. (1-14) will be hereafter considered as the postmaneuver state specified by either $(\delta\boldsymbol{R}_i^+,\delta\boldsymbol{V}_i^+)$ or $(\delta\boldsymbol{R}_i,\delta\boldsymbol{V}_i^+)$ since $\delta\boldsymbol{R}_i^+=\delta\boldsymbol{R}_i$.

Most textbooks define the guidance problem as one of finding the maneuver that compensates for an initial position perturbation. A more general approach is to consider the maneuvers as a function of the initial perturbations in position, velocity, and time

$$\delta\boldsymbol{V}_m=G_R\delta\boldsymbol{R}_i+G_V\delta\boldsymbol{V}_i+G_T\delta t_i$$

$$=\frac{\partial\boldsymbol{V}_m}{\partial\boldsymbol{R}_i}\delta\boldsymbol{R}_i+\frac{\partial\boldsymbol{V}_m}{\partial\boldsymbol{V}_i}\delta\boldsymbol{V}_i+\frac{\partial\boldsymbol{V}_m}{\partial t_i}\delta t_i \tag{21}$$

where G_R and G_V are the position and velocity guidance matrices, and $\boldsymbol{G}_t$ the time-guidance vector. These guidance matrices and guidance vector will collectively be referred to as the guidance parameters. Imposing the required number of guidance constraints allows the guidance problem to be solved for the postmaneuver velocity

$$\delta\boldsymbol{V}_i^+=G_R^+\delta\boldsymbol{R}_i+G_V^+\delta\boldsymbol{V}_i+\boldsymbol{G}_T^+\delta t_i \tag{22}$$

Comparing Eqs. (20-22),

$$G_R^+\delta\boldsymbol{R}_i+G_V^+\delta\boldsymbol{V}_i+\boldsymbol{G}_T^+\delta t_i-\delta\boldsymbol{V}_i=G_R\delta\boldsymbol{R}_i+G_v\delta\boldsymbol{V}_i+\boldsymbol{G}_T\delta t_i \tag{23}$$

Since $\delta\boldsymbol{R}_i$, $\delta\boldsymbol{V}_i$, and δt_i are independent and arbitrary,

$$G_R=G_R^+,\quad G_V=G_V^+-I_3,\quad \boldsymbol{G}_T=\boldsymbol{G}_T^+ \tag{24}$$

Because the time-guidance vector $\boldsymbol{G}_T$ is a function of $\delta\boldsymbol{X}_i$ and δt_i, it can be related to the position and velocity guidance matrices by inserting $\delta\boldsymbol{V}_m$ for $\boldsymbol{Y}$ in the equivalence relationship, Eq. (7),

$$\boldsymbol{G}_T=-G_R\boldsymbol{V}_i-G_V\boldsymbol{A}_i \tag{25}$$

Based on Eqs. (22-24), and assuming that the maneuver has no effect on the final time at the next maneuver point (i.e., fixed-time-of-arrival guidance), the position-velocity-time state can be updated through the maneuver using the seven-dimensional matrix update equation

$$\begin{bmatrix}\delta\boldsymbol{X}_i^+\\ \delta t_i^+\end{bmatrix}=\begin{bmatrix}G_6 & G_{T6}\\ \boldsymbol{0}_6^T & -1\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{X}_i\\ \delta t_i\end{bmatrix}=G_{\mathrm{FT}}\begin{bmatrix}\delta\boldsymbol{X}_i\\ \delta t_i\end{bmatrix}$$

$$\begin{bmatrix}\delta\boldsymbol{R}_i^+\\ \delta\boldsymbol{V}_i^+\\ \delta t_i^+\end{bmatrix}=\begin{bmatrix}I_3 & 0_3 & \boldsymbol{0}\\ G_R & I_3+G_V & \boldsymbol{G}_T\\ \boldsymbol{0}^T & \boldsymbol{0}^T & -1\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{R}_i\\ \delta\boldsymbol{V}_i\\ \delta t_i\end{bmatrix}=G_{\mathrm{FT}}\begin{bmatrix}\delta\boldsymbol{R}_i\\ \delta\boldsymbol{V}_i\\ \delta t_i\end{bmatrix} \tag{26}$$

where

$$\delta\boldsymbol{X}_i^+=\begin{bmatrix}\delta\boldsymbol{R}_i^+\\ \delta\boldsymbol{V}_i^+\end{bmatrix},\quad G_{T6}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{G}_T\end{bmatrix},\quad G_6=\begin{bmatrix}I_3 & 0_3\\ G_R & I_3+G_V\end{bmatrix}$$

0_3 is a 3×3 null matrix. Since the final point is to remain fixed in time, δt_i^+ will represent the change in update time to the final point [i.e., δt_i^+ represents δt in Eq. (2)]. G_{FT} is referred to as the (7×7) fixed-time-of-arrival (FTA) guidance matrix.

For those maneuvers that have an effect on the flight time following the maneuver, the guidance matrix has to include the final time-guidance partials $\boldsymbol{T}_R$, $\boldsymbol{T}_V$, and T_T

$$\delta t_f=\boldsymbol{T}_R^T\delta\boldsymbol{R}_i+\boldsymbol{T}_V^T\delta\boldsymbol{V}_i+T_T\delta t_i$$

$$=\frac{\partial t_f^T}{\partial\boldsymbol{R}_i}\delta\boldsymbol{R}_i+\frac{\partial t_f^T}{\partial\boldsymbol{V}_i}\delta\boldsymbol{V}_i+\frac{\partial t_f}{\partial t_i}\delta t_i \tag{27}$$

The time partials can also be expressed in terms of the postmaneuver conditions

$$\delta t_f=\boldsymbol{T}_R^{+T}\delta\boldsymbol{R}_i^++\boldsymbol{T}_V^{+T}\delta\boldsymbol{V}_i^++T_T^+\delta t_i^+ \tag{28}$$

The final time-guidance partial T_T can be related to the final time-guidance partial vectors $\boldsymbol{T}_R$ and $\boldsymbol{T}_V$. Substituting δt_f for $\boldsymbol{Y}$ in the time-state equivalence relationship, Eq. (7),

$$T_T=-\boldsymbol{T}_R^T\boldsymbol{V}_i-\boldsymbol{T}_V^T\boldsymbol{A}_i \tag{29}$$

Inserting the guidance-time partials into the bottom row of the FTA guidance matrix, Eq. (26), results in the (7×7) variable-time-of-arrival (VTA) guidance matrix G_{VT}

$$\begin{bmatrix}\delta\boldsymbol{X}_i^+\\ \delta t_i^+\end{bmatrix}=\begin{bmatrix}G_6 & G_{T6}\\ \boldsymbol{T}_X^T & T_T^{-1}\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{X}_i\\ \delta t_i\end{bmatrix}=G_{\mathrm{VT}}\begin{bmatrix}\delta\boldsymbol{X}_i\\ \delta t_i\end{bmatrix}$$

$$\begin{bmatrix}\delta\boldsymbol{R}_i^+\\ \delta\boldsymbol{V}_i^+\\ \delta t_i^+\end{bmatrix}=\begin{bmatrix}I_3 & 0_3 & \boldsymbol{0}\\ G_R & I_3+G_V & \boldsymbol{G}_T\\ \boldsymbol{T}_R^T & \boldsymbol{T}_V^T & T_T-1\end{bmatrix}\begin{bmatrix}\delta\boldsymbol{R}_i\\ \delta\boldsymbol{V}_i\\ \delta t_i\end{bmatrix}=G_{\mathrm{VT}}\begin{bmatrix}\delta\boldsymbol{R}_i\\ \delta\boldsymbol{V}_i\\ \delta t_i\end{bmatrix} \tag{30}$$

where $\boldsymbol{T}_X$ is the vector $(\boldsymbol{T}_R,\boldsymbol{T}_V)$. The time perturbation δt_i^+ following the maneuver is assumed to be the time perturbation involved in updating the state to the final point

$$\delta t_i^+=\delta t_f-\delta t_i \tag{31}$$

In those areas where there is to be an update of the state vector based on the nominal time plus a time equal to the initial time perturbation (i.e., holding the final nominal time fixed), the FTA guidance matrix G_{FT} can be updated through the transition matrix Φ_A, Eq. (2),

$$\begin{bmatrix} \delta X_f \\ \delta t \end{bmatrix} = \Phi_A G_{\rm FT} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = G_{\rm TF} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = \begin{bmatrix} \Phi G_6 & \Phi G_{T6} - D_f \\ 0^T & -1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = \begin{bmatrix} \Phi_1 + \Phi_2 G_R & \Phi_2 (I_3 + G_V) & \Phi_2 G_T - V_f \\ \Phi_3 + \Phi_4 G_R & \Phi_4 (I_3 + G_V) & \Phi_4 G_T - A_f \\ 0^T & 0^T & -1 \end{bmatrix} \begin{bmatrix} \delta R_i \\ \delta V_i \\ \delta t_i \end{bmatrix} \tag{32}$$

where δt is the change in flight time following the maneuver, resulting in arrival at the final point at the nominal final time. $G_{\rm TF}$ is referred to as the (7×7) FTA time-guidance update matrix.

When a perturbation in final flight time exists, multiplying $G_{\rm VT}$ by Φ_A results in

$$\begin{bmatrix} \delta X_f \\ \delta t \end{bmatrix} = \Phi_A G_{\rm VT} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = G_{\rm TV} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = \begin{bmatrix} \Phi G_6 + D_f T_X^T & \Phi G_{T6} + D_f (T_T - 1) \\ T_X^T & T_T - 1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_1 + \Phi_2 G_R + V_f T_R^T & \Phi_2 (I_3 + G_V) + V_f T_V^T & \Phi_2 G_T + V_f (T_T - 1) \\ \Phi_3 + \Phi_4 G_R + A_f T_R^T & \Phi_4 (I_3 + G_V) + A_f T_V^T & \Phi_4 G_T + A_f (T_T^{-1}) \\ T_R^T & T_V^T & T_T - 1 \end{bmatrix} \begin{bmatrix} \delta R_i \\ \delta V_i \\ \delta t_i \end{bmatrix} \tag{33}$$

where δt is the change in flight time following the maneuver, and δX_f is defined with respect to the nominal final point. The matrix $G_{\rm TV}$ is referred to as the (7×7) VTA time-guidance update matrix. If δt is to represent δt_f, then the lower right term in $G_{\rm TV}$ should be just T_T.

The time-guidance update matrix for variable-time-of-arrival guidance, based on a final state perturbation defined with respect to a target trajectory (subscript t), can be obtained by multiplying $G_{\rm VT}$ by Φ_A. However this result must be modified by subtracting D_t from the first six rows of the last column since, for instance, the desired position perturbation due to an initial time perturbation must consist of the terms

$$(\Phi_2 G_T - V_f + V_{fr} T_T)\delta t_i = \Phi_2 \delta V_m - V_f \delta t_i + V_{fr} \delta t_f \tag{34}$$

The resulting time-guidance update matrix is

$$\begin{bmatrix} \delta X_{fr} \\ \delta t \end{bmatrix} = G_{\rm TVR} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix} = \begin{bmatrix} \Phi G_6 + D_{fr} T_X^T & \Phi G_{T6} + D_{fr} (T_T - 1) - D_t \\ T_X^T & T_T - 1 \end{bmatrix} \begin{bmatrix} \delta X_i \\ \delta t_i \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_1 + \Phi_2 G_R + V_{fr} T_R^T & \Phi_2 (I_3 + G_V) + V_{fr} T_V^T & \Phi_2 G_T + V_{fr} (T_T - 1) - V_t \\ \Phi_3 + \Phi_4 G_R + A_{fr} T_R^T & \Phi_4 (I_3 + G_V) + A_{fr} T_V^T & \Phi_4 G_T + A_{fr} (T_T - 1) - A_t \\ T_R^T & T_V^T & T_T - 1 \end{bmatrix} \begin{bmatrix} \delta R_i \\ \delta V_i \\ \delta t_i \end{bmatrix} \tag{35}$$

where δt is the change in flight time. $G_{\rm TVR}$ is considered the relative VTA time-guidance update matrix.

Solving the Linear Guidance Problem

There are two basic methods of solving the linearized guidance problem. One way, referred to as the direct guidance method, is based on updating the initial perturbed state, modified by the maneuver, to the terminal point at time $t_f + \delta t_f$

$$\delta R_f = \Phi_1 \delta R_i + \Phi_2 (\delta V_i + \delta V_m) - V_f \delta t_i + V_{fr} \delta t_f \tag{36}$$

Equation (36) must be combined with the guidance constraints, which are generally imposed on the maneuver and/or the final position perturbation, and solved for the maneuver in terms of the initial state perturbations. The time-guidance update matrices, Eqs. (32), (33), and (35), are useful in the direct guidance method since they include the effects of the maneuver.

The other method, referred to as the offset guidance method, is based on the principle of linear superposition. A trajetory based on the nominal trajectory plus the initial state perturbations in the nominal may be extrapolated to the nominal final time t_f, obtaining

$$\delta R_{fi} = \Phi_1 \delta R_i + \Phi_2 \delta V_i - V_f \delta t_i \tag{37}$$

A trajectory may also be extrapolated to the same time based on the nominal trajectory plus the desired maneuver, obtaining

$$\delta R_{fm} = \left[\frac{\partial V_m}{\partial R_{fm}}\right]^{-1} \delta V_m, \quad \delta V_m = \frac{\partial V_m}{\partial R_{fm}} \delta R_{fm} \tag{38}$$

Because the maneuver is to compensate for the initial state errors, the principle of linear superposition can be used to establish that at the nominal arrival time: 1) the two extrapolations are equal in magnitude and opposite in direction for fixed-time-of-arrival guidance (Fig. 2), and 2) the two extrapolations differ by the relative motion $-V_{fr}\delta t_f$ (assuming a target vehicle) for variable-time-of-arrival guidance (Fig. 3). The difference between the nominal and the final perturbed trajectory is $-V_{fr}\delta t_f$ at both the nominal time of flight and at time $t_f + \delta t_f$ (see Fig. 3). For fixed- and variable-time-of arrival guidance, respectively, the principle of linear superposition therefore yields

$$\delta R_{fi} + \delta R_{fm} = 0, \qquad \delta R_{fi} + \delta R_{fm} = -V_{fr} \delta t_f \tag{39}$$

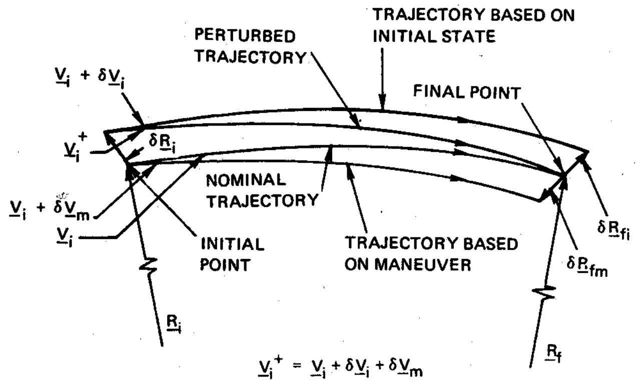

Fig. 2 Use of offset guidance method to solve FTA guidance problem.

A special guidance matrix can be defined by

$$\delta V_m = -G_{RF}\delta R_{fi} \tag{40}$$

For fixed-time-of-arrival guidance, this guidance matrix can also be expressed as

$$\delta V_m = G_{RF}\delta R_{fm} \tag{41}$$

The matrix G_{RF} is considered the fundamental guidance matrix because once found the other guidance partials, G_R and G_V, and the guidance vector, G_T, can easily be obtained by combining Eqs. (21), (37), and (40):

$$G_R = -G_{RF}\Phi_1, \quad G_V = -G_{RF}\Phi_2, \quad G_T = G_{RF}V_f \tag{42}$$

Eliminating G_{RF} between the relationships in Eq. (42) yields

$$G_R = G_V\Phi_2^{-1}\Phi_1, \quad G_V = G_R\Phi_1^{-1}\Phi_2$$
$$G_T = -G_R\Phi_1^{-1}V_f = -G_V\Phi_2^{-1}V_f \tag{43}$$

The FTA guidance matrix G_{FT} can be expressed in terms of the fundamental position guidance partial G_{RF} [see Eq. (26)]:

$$G_{FT} = \begin{bmatrix} I_3 & 0_3 & 0 \\ -G_{RF}\Phi_1 & I_3 - G_{RF}\Phi_2 & G_{RF}V_f \\ 0^T & 0^T & -1 \end{bmatrix} \tag{44}$$

The FTA time-guidance update matrix G_{TF} can also be expressed in terms of G_{RF} [see Eq. (32)]:

$$G_{TF} = \begin{bmatrix} \Phi_1 - \Phi_2 G_{RF}\Phi_1 & \Phi_2(I_3 - G_{RF}\Phi_2) & \Phi_2 G_{RF}V_f - V_f \\ \Phi_3 - \Phi_4 G_{RF}\Phi_1 & \Phi_4(I_3 - G_{RF}\Phi_2) & \Phi_4 G_{RF}V_f - A_f \\ 0^T & 0^T & -1 \end{bmatrix} \tag{45}$$

The guidance laws considered herein are linearized using both direct and offset guidance methods for comparison purposes. The offset method is new, whereas the direct method has been employed in the past. The analysis contained herein is restricted to guidance constraints that involve the time, maneuver, and terminal position perturbation. Since all Shuttle guidance laws and apparently all guidance laws presented in the literature deal only with these quantities, this approach applies to all known guidance problems. The approach can be generalized to allow constraints on the final velocity perturbation.

Fixed-Time-of-Arrival Guidance

Fixed-time-of-arrival guidance is frequently defined as implying not only a constant time of flight but also a postmaneuver trajectory that passes through the final nominal point. This form of FTA guidance is sometimes referred to as Lambert guidance to distinguish it from other FTA guidance laws such as the Shuttle phasing and height maneuvers. These maneuvers are not constrained to coincide with the final nominal point.

The offset guidance approach to deriving the guidance laws associated with Lambert guidance involves the determination of the fundamental guidance matrix G_{RF} from the guidance constraints. Since the guidance constraints are such that δR_f should be zero at the final nominal time, the matrix G_{RF} can be obtained using the model of linearized motion represented by either the time or Lambert transition matrices [see Eq. (15)]:

$$\delta R_f = \frac{\partial R_f}{\partial R_i}\delta R_i + \frac{\partial R_f}{\partial V_i^+}\delta V_i^+ = \Phi_1\delta R_i + \Phi_2\delta V_i^+$$
$$\delta V_i^+ = \frac{\partial V_i^+}{\partial R_i}\delta R_i + \frac{\partial V_i^+}{\partial R_f}\delta R_f = -\Phi_2^{-1}\Phi_1\delta R_i + \Phi_2^{-1}\delta V_i \tag{46}$$

Setting δR_f equal to zero in either of Eqs. (46) results in

$$G_{RF} = \frac{\partial V_i^+}{\partial R_f} = \left[\frac{\partial R_f}{\partial V_i^+}\right]^{-1} = \Phi_2^{-1} \tag{47}$$

The guidance partials can be obtained by combining Eqs. (42) and (47).

To derive the Lambert guidance law using the direct guidance approach, the FTA time-guidance update matrix, Eq. (32), can be used since this matrix was based on a guidance matrix that contained the effects of the maneuver. Because the postmaneuver trajectory passes through the final nominal point at the final time, all three terms in the upper three rows of Eq. (32) can be set equal to zero to obtain the Lambert guidance laws

$$G_R = -\Phi_2^{-1}\Phi_1$$
$$G_V = -\Phi_2^{-1}\Phi_2 = -I_3, \quad G_V^+ = 0_3$$
$$G_T = \Phi_2^{-1}V_f \tag{48}$$

in agreement with the results from the offset guidance approach. The Lambert maneuver δV_m is found using Eq. (21)

$$\delta V_m = -\Phi_2^{-1}\Phi_1\delta R_i - \delta V_i + \Phi_2^{-1}V_f\delta t_i \tag{49}$$

One way of deriving the time-guidance vector is to convert the initial time perturbation into initial perturbations in position and velocity [see Eq. (10)]

$$\delta R_i = -V_i\delta t_i, \quad \delta V_i = -A_i\delta t_i \tag{50}$$

The maneuver, based on perturbations in δR_i and δV_i, is given by Eq. (49) [see Eq. (8)]

$$\delta V_m = G_R\delta R_i + G_V\delta V_i = (\Phi_2^{-1}\Phi_1 V_i + A_i)\delta t_i$$
$$= \Phi_2^{-1}(\Phi_1 V_i + \Phi_2 A_i)\delta t_i = \Phi_2^{-1}V_f\delta t_i = G_T\delta t_i \tag{51}$$

The derivative of the position guidance partial G_R can be obtained with the aid of Eq. (11)

$$\frac{dG_R}{dt} = \Phi_2^{-1}\Phi_2^{-T} \tag{52}$$

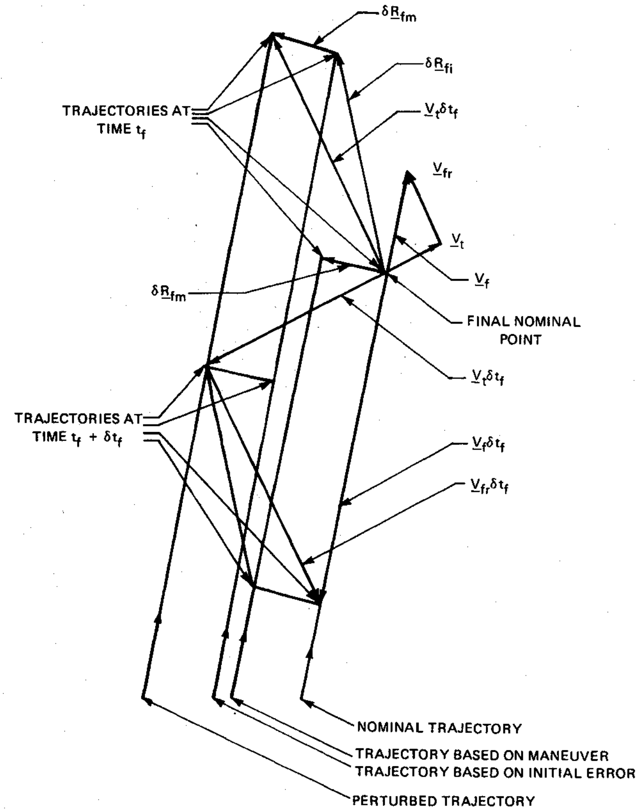

Fig. 3 Linearized motion at final point for VTA guidance with negative δt_f.

Since the initial condition for the integration of Eq. (52) involves a matrix containing infinite terms (because an infinite velocity is required for an instantaneous correction of a position perturbation), the derivative of the inverse matrix can be used:

$$\frac{\mathrm{d}G_R^{-1}}{\mathrm{d}t}=\Phi_1^{-1}\Phi_1^{-T} \tag{53}$$

which shows that G_R must be symmetrical since the initial condition for G_R is a null matrix.

Battin[10] presents a different differential equation for G_R, which was originally referred to in the literature as the Q matrix (Battin uses the symbol C^*)

$$\frac{\mathrm{d}G_R^{-1}}{\mathrm{d}t}=I_3-G_R^{-1}G_G G_R^{-1} \tag{54}$$

Equation (54) must be integrated backward along the trajectory with the initial condition of a null matrix.

Variable-Time-of-Arrival Guidance

Variable-time-of-arrival guidance involves selecting a time for the terminal point on the perturbed trajectory that differs from the nominal final time. Frequently, the state perturbation is constrained to intercept a target trajectory that passes through the final nominal point. The perturbation in flight time is selected to minimize the maneuver magnitude. The offset guidance approach is based on combining Eq. (38) and the second equation in Eq. (39)

$$\delta V_m=\frac{\partial V_m}{\partial R_{fm}}\delta R_m=-\Phi_2^{-1}(\delta R_{fi}+V_{fr}\delta t_f) \tag{55}$$

where Φ_2^{-1} has been substituted for $\partial V_m/\partial R_{fm}$ since there are no constraints imposed on the maneuver. Because δt_f is going to be a function of the initial state errors that appear in δR_{fi}, Eq. (55) cannot be used to justify setting the G_{RF} matrix equal to Φ_2^{-1}. Minimizing the maneuver with respect to δt_f can be accomplished by differentiation and setting the result equal to zero, or by subtracting from δV_m the component of δV_m which lies parallel to the vector $\Phi_2^{-1}V_{fr}$. Solving for δt_f and combining with Eq. (55) yields

$$\delta V_m=G_{\mathrm{Tr}}\Phi_2^{-1}\delta R_{fi} \tag{56}$$

where

$$G_{\mathrm{Tr}}=I_3-G_{\mathrm{Tr}}G_{\mathrm{Tr}}^T/d,\quad G_{\mathrm{Tr}}=\Phi_2^{-1}V_{fr},\quad d=G_{\mathrm{Tr}}^TG_{\mathrm{Tr}}$$

The fundamental guidance partial for VTA guidance is [see Eq. (38)]

$$G_{\mathrm{RF}}=G_{\mathrm{Tr}}=\Phi_2^{-1} \tag{57}$$

When the target and perturbed trajectories are to intersect at rendezvous, the direct guidance approch used to obtain the guidance partials is defined by setting the terms in the relative VTA time-guidance update matrix, Eq. (33), equal to zero, resulting in the interrelations between the guidance laws and the guidance time partials

$$\begin{aligned}
G_R&=-\Phi_2^{-1}(V_{fr}T_R^T+\Phi_1), & T_R&=-(G_R^T\Phi_2^T+\Phi_1^T)V_{fr}/c\\
G_V&=-\phi_2^{-1}(V_{fr}T_V^T+\Phi_2), & T_V&=-(G_V^T\Phi_2^T+\Phi_2^T)V_{fr}/c\\
G_T&=-\phi_2^{-1}(V_{fr}T_T-V_f), & T_T&=-(G_T^T\Phi_2^T-V_f^T)V_{fr}/c
\end{aligned} \tag{58}$$

where

$$c=V_{fr}^TV_{fr}$$

Whenever the fundamental guidance matrix satisfies Eq. (42), Eq. (58) can be reduced to

$$\begin{aligned}
T_R&=\Phi_1^T(G_{\mathrm{RF}}^T\Phi_2^T-I_3)V_{fr}/c\\
T_V&=\Phi_2^T(G_{\mathrm{RF}}^T\Phi_2^T-I_3)V_{fr}/c\\
T_T&=-V_F^T(G_{\mathrm{RF}}^T\Phi_2^T-I_3)V_{fr}/c
\end{aligned} \tag{59}$$

The direct guidance approach to VTA guidance defines a final position perturbation relative to a target trajectory by propagating the postmaneuver state perturbations δR_i, δV_i^+, and δt_i, and then adding a term to introduce the relative motion with respect to the target trajectory during the time δt_f

$$\delta R_f=\Phi_1\delta R_i+\Phi_2\delta V_i^+-V_f\delta t_i+V_{fr}\delta t_f \tag{60}$$

Setting δR_f equal to zero and solving for δV_i^+ yields

$$\delta V_i^+=-\Phi_2^{-1}\Phi_1\delta R_i+\Phi_2^{-1}V_f\delta t_i-\Phi_2^{-1}V_{fr}\delta t_f \tag{61}$$

Introducing the FTA position guidance matrix G_{RFT} and the time-guidance vector G_{TFT} [see Eq. (48)] allows Eq. (61) to be expressed as

$$\delta V_i^+=\delta V_{\mathrm{FG}}^+-G_{\mathrm{Tr}}\delta t_f \tag{62}$$

where

$$\begin{aligned}
\delta V_{\mathrm{FG}}^+&=G_{\mathrm{RFT}}\delta R_i+G_{\mathrm{TFT}}\delta t_i\\
G_{\mathrm{RFT}}&=-\Phi_2^{-1}\Phi_1\\
G_{\mathrm{TFT}}&=-\Phi_2^{-1}V_f\\
G_{\mathrm{Tr}}&=\Phi_2^{-1}V_{fr}
\end{aligned}$$

The equation

$$\frac{\partial\,|\delta V_i^+ - \delta V_i|}{\partial t_f} = \frac{\partial\,|\delta V_m|}{\partial t_f} = 0 \tag{63}$$

defines the minimization of the maneuver magnitude. Minimizing the function in Eq. (63) with respect to δt_f, and then solving for δt_f yields the time-guidance partials

$$T_R = G_{\mathrm{RFT}} G_{\mathrm{Tr}}/d = -\Phi_2^{-1}\Phi_1\Phi_2^{-1}V_{fr}/d$$

$$T_V = G_{\mathrm{VFT}} G_{\mathrm{Tr}}/d = -\Phi_2^{-1}\Phi_2\Phi_2^{-1}V_{fr}/d = -G_{\mathrm{Tr}}/d$$

$$T_T = G_{\mathrm{TFT}}^T G_{\mathrm{Tr}}/d = V_f^T\Phi_2^{-T}\Phi_2^{-1}V_{fr}/d \tag{64}$$

where d is the magnitude defined in Eq. (56) and G_{VFT} is the FTA velocity guidance matrix [see Eq. (48)]

$$G_{\mathrm{VFT}} = -I_3 \tag{65}$$

All three time-guidance partials are obtained by multiplying the vector G_{Tr} by the corresponding FTA guidance law, and then dividing by d. Note that the time-guidance partials satisfy the equivalence principle represented by Eq. (7).

The time partials can be inserted into Eq. (62) using Eq. (27)

$$\delta V_i^+ = G_{\mathrm{Tr}}\delta V_{\mathrm{FG}}^+ + (I_3 - G_{\mathrm{Tr}})\delta V_i \tag{66}$$

where G_{Tr}, a matrix projection operator, is defined in Eq. (56). The corresponding maneuver δV_m for VTA guidance is

$$\delta V_m = \delta V_i^+ - \delta V_i = G_{\mathrm{Tr}}(\delta V_{\mathrm{FG}}^+ - \delta V_i) \tag{67}$$

where δV_m is perpendicular to the vector G_{Tr}. There will be no maneuver required when the initial perturbations satisfy the relationship

$$\delta V_i = \delta V_{\mathrm{FG}}^+ = -\Phi_2^{-1}(\Phi_1\delta R_i - V_f\delta t_i) \tag{68}$$

The guidance laws are obtained by comparing Eqs. (21), (24), and (67):

$$G_R = G_{\mathrm{Tr}} G_{\mathrm{RFT}}$$

$$G_V = G_{\mathrm{Tr}} G_{\mathrm{VFT}} = -G_{\mathrm{Tr}}, \quad G_V^+ = I_3 - G_{\mathrm{Tr}}$$

$$G_T = G_{\mathrm{Tr}} G_{\mathrm{TFT}} \tag{69}$$

All three of the FTA guidance laws are multiplied by G_{Tr} in order to obtain the corresponding VTA guidance law. The matrices G_R and G_V are symmetrical. Inserting the definitions for G_{VFT}, Eq. (65), and the FTA parameters, Eq. (62), into Eq. (69) results in

$$G_R = -\Phi_2^{-1}(I_3 - V_{fr}V_{fr}^T\Phi_2^{-T}\Phi_2^{-1}/d)\Phi_1 = G_{\mathrm{RF}}\Phi_1$$

$$G_V = -\Phi_2^{-1}(I_3 - V_{fr}V_{fr}^T\Phi_2^{-T}\Phi_2^{-1}/d)\Phi_2 = G_{\mathrm{RF}}\Phi_2$$

$$G_T = \Phi_2^{-1}(I_3 - V_{fr}V_{fr}^T\Phi_2^{-T}\Phi_2^{-1}/d)V_f = -G_{\mathrm{RF}}V_f \tag{70}$$

Because a vector multiplied by a matrix projection operator based on the same vector is a null vector, the following conditions apply to the velocity time-guidance partial T_V

$$G_R^T T_V = 0, \quad G_V^T T_V = 0, \quad G_T^T T_V = 0, \quad G_{\mathrm{RF}}^T T_V = 0 \tag{71}$$

Table 1 Summary of FTA and VTA guidance laws

Guidance category	Guidance laws
Fixed-time-of-arrival guidance	$G_{\mathrm{RF}} = \Phi_2^{-1} = G_{\mathrm{RFFT}}$ $G_R = -G_{\mathrm{RF}}\Phi_1 = -\Phi_2^{-1}\Phi_1 = G_{\mathrm{RFT}}$ $G_V = -G_{\mathrm{RF}}\Phi_2 = -\Phi_2^{-1}\Phi_2 = G_{\mathrm{VFT}}$ $G_T = = G_{\mathrm{RF}}V_f = \Phi_2^{-1}V_f = G_{\mathrm{TFT}}$
Variable-time-of arrival guidance	$G_{\mathrm{RF}} = G_{\mathrm{Tr}}G_{\mathrm{RFFT}} = G_{\mathrm{Tr}}\Phi_2^{-1}$ $T_R = G_{\mathrm{RFT}}G_{\mathrm{Tr}}/d = \Phi_1^T(G_{\mathrm{RF}}^T\Phi_2^T - I_3)V_{fr}/c$ $T_V = G_{\mathrm{VFT}}G_{\mathrm{Tr}}/d = \Phi_2^T(G_{\mathrm{RF}}^T\Phi_2^T - I_3)V_{fr}/c$ $T_T = G_{\mathrm{TFT}}G_{\mathrm{Tr}}/d = -V_f^T(G_{\mathrm{RF}}^T\Phi_2^T - I_3)V_{fr}/c$ $G_R = G_{\mathrm{Tr}}G_{\mathrm{RFT}} = -G_{\mathrm{RF}}\Phi_1$ $G_V = G_{\mathrm{Tr}}G_{\mathrm{VFT}} = -G_{\mathrm{RF}}\Phi_2$ $G_T = G_{\mathrm{Tr}}G_{\mathrm{TFT}} = G_{\mathrm{RF}}V_f$

$G_{\mathrm{Tr}} = I_3 - G_{\mathrm{Tr}}G_{\mathrm{Tr}}^T/d$, $G_{\mathrm{Tr}} = \Phi_2^{-1}V_{fr}$, $V_{fr} = V_f - V_t$, $c = V_{fr}^T V_{fr}$, $d = G_{\mathrm{Tr}}^T G_{\mathrm{Tr}}$.

Shuttle Phasing and Height Maneuver Guidance

Two of the maneuvers involved in typical Shuttle rendezvous profiles are phasing and height maneuvers. These maneuvers are constrained to be horizontal maneuvers. They differ only in their final position constraints. The phasing maneuver terminates on the plane that contains the final nominal position vector and is normal to the orbital plane. The height maneuver terminates on the horizontal plane at the final point. Frequently, both the phasing and height maneuvers will be contained in the same rendezvous sequence. The first maneuver in the sequence is usually followed by an update through half of an orbital period, with the second maneuver followed by an update to a given time.

The offset guidance approach to deriving guidance laws for phasing/height guidance involves directly applying the guidance constraints to obtain the fundamental guidance matrix G_{RF}. The maneuver is obtained by multiplying the unit vector U_{hi} in the horizontal direction by the magnitude of the maneuver δV_{hi}. This maneuver can then be propagated along the trajectory using the submatrix Φ_2 from the transition matrix, which extends from the maneuver point to the point at which the phasing or height constraint is imposed. For an "NREV" transition matrix with N equal to 0.5, Eq. (19) can be used after finding the time partial T by differentiating the period of the orbit, multiplied by 0.5, with respect to the state. The desired component of the final position vector is then obtained by dotting the updated position vector with the normal to the plane direction K

$$K^T\Phi_2 U_{hi}\delta V_{hi} = K^T\delta R_{fm} \tag{72}$$

where

$K = \mathrm{unit}(R_f)$ for height maneuver

$= \mathrm{unit}((R_f \times V_f) \times R_f)$ for phasing maneuver

Solving Eq. (72) for δV_{hi} and using the maneuver definition

$$\delta V_m = U_{hi}\delta V_{hi} = U_{hi}K^T\delta R_{fm}/K^T\Phi_2 U_{hi} \tag{73}$$

allows the fundamental guidance partial to be obtained by comparing Eqs. (41) and (73)

$$G_{\mathrm{RF}} = U_{hi}K^T/(K^T\Phi_2 U_{hi}) \tag{74}$$

Equation (74) can be combined with Eq. (42) to obtain the guidance partials.

The derivation of the phasing and height guidance laws by the direct guidance method starts with the update of the perturbed state including the maneuver.

$$\delta R_f = \Phi_1\delta R_i + \Phi_2\delta V_i^+ - V_f\delta t_i \tag{75}$$

where Φ_1 and Φ_2 are from the transition matrix which extends from the maneuver point to the point at which the phasing or height constraint is imposed. The constraint that the manevuer must be horizontal dictates that

$$\delta V_i^+ = U_{hi}\delta V_{hi} + \delta V_i \tag{76}$$

The constraint that the final point must lie in a specified plane can be expressed as

$$K^T \delta R_f = 0 \tag{77}$$

where K is defined in Eq (72). Combining Eqs. (75-77),

$$K^T [\Phi_1 \delta R_i + \Phi_2 (U_{hi}\delta V_{hi} + \delta V_i) - V_f \delta t_i] = 0 \tag{78}$$

Solving for δV_{hi},

$$\delta V_{hi} = -K^T (\Phi_1 \delta R_i + \Phi_2 \delta V_i - V_f \delta t_i)/(K^T \Phi_2 U_{hi}) \tag{79}$$

Since the maneuver equals $U_{hi}\delta V_{hi}$,

$$\delta V_m = -U_{hi} K^T (\Phi_1 \delta R_i + \Phi_2 \delta V_i - V_f \delta t_i)/(K^T \Phi_2 U_{hi}) \tag{80}$$

which leads to the same guidance laws as obtained with G_{RF}

$$G_R = -U_{hi} K^T \Phi_1 / (K^T \Phi_2 U_{hi})$$

$$G_V = -U_{hi} K^T \Phi_2 / (K^T \Phi_2 U_{hi})$$

$$G_T = U_{hi} K^T V_f / (K^T \Phi_2 U_{hi}) \tag{81}$$

Note that the guidance laws are interrelated according to the equivalence principle, Eq. (7).

Conclusions

Linearized guidance laws define an impulsive maneuver which nulls the effects of initial state perturbations in position, velocity, and time. The fixed-time-of-arrival linearized guidance problem can be formulated in terms of two guidance matrices, G_R and G_V, and one guidance vector, G_T. These quantities must be supplemented by two final time partial vectors, T_R and T_V, and one scalar, T_T, when solving the variable-time-of-arrival guidance problem. The guidance parameters G_R, G_V, G_T, T_R, T_V, and T_T can all be expressed as a function of a single fundamental guidance matrix G_{RF}. The guidance parameters can be inserted into guidance matrices, which allow the initial state perturbations to be updated through the maneuver. When the guidance matrices are multiplied by the augmented time transition matrix, the time-guidance update matrices are obtained which propagate the initial state perturbation to the final desired point. Table 1 summarizes the fixed- and variable-time-of-arrive guidance laws and their interrelations.

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